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Small Line Power Transformers with Minimized Losses

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1. Introduction

In spite of the rapid development and implementation of high-frequency transformers in the network power supplies with low and middle powers, line power frequency transformers, which are used for the same purpose, still have significant importance. The latter are realized with common materials and are more reliable. Further down in the statement these transformers are termed for short as: "small transformers"

Small transformers are comprised of many thin lamellas or of winding silicon steel strip made from high-quality silicon steel.

Usually the main criteria accepted when designing small transformers is the achieving of maximum power of the transformer, while keeping the costs as little as possible, when the transformer is acceptably heated. As the savings of electrical power have become more and more important, this article strives to find an acceptable compromise between lowering the losses of electrical power and increasing the price of the transformer.

In order to examine the profitability of small transformers with reduced losses a comparison is done between the maximum power and consequently minimal price type (termed as "base transformer") and the one with reduced energy losses.

2. Used symbols

The following symbols will be used further on:

E – inducted electric voltage in the winding, V;
I_{ef} – effective electric current in the winding, A;
F – line power frequency, Hz;
P_{tr} – transformer power, W;
P_{tot} – transformer total losses, W;
P_{Fe} – core losses, W;

P_{dFe} – specific core losses, W/kg;
P_{Cu} – losses in the windings, W;
T – transformer temperature, °C;
ΔT – transformer overtemperature, °C;
R_{th} – transformer thermal resistance, °C per 1 W;
$\hat{B}$ – magnetic induction maximum value, T;
V_{Fe} – core volume, cm^3;
l_{Fe} – average magnetic length, cm;
A_{Fe} – core effective cross section, cm^2;
g_{Fe} – core specific weight, kg/cm^3;
S_w – cross section of the winding window, mm^2;
l_{Cu} – average winding length, m;
w – number of winding turns;
r – winding DC resistance, Ω;
ρ_{Cu} – copper wire specific resistance, (Ω.mm^2)/m;
q_{Cu} – winding wire cross section, mm^2;
k_{Cu} – filling coefficient of S_w with pure copper for one winding;
k_c – filling coefficient of core cross section.

3. Theoretical standing

The study has been carried out with the following preconditions present:

- sinusoidal supply voltage;
- primary and secondary windings of the transformer with equal number of turns;
- equal value of effective electric current (I_{ef}) in both windings;
- The reactance leakage of the windings is neglected.

The transformer's total losses (P_{tot}) are a sum of the core losses (P_{Fe}) and the losses in the windings (P_{Cu}), i.e.:

(1) $$P_{tot} = P_{Fe} + P_{Cu}.$$

The losses in the windings are represented by the formula:

(2) $$P_{Cu} = 2rI_{ef}^{\,2}.$$

When taking into account that:

$$r = \frac{\rho_{Cu} l_{Cu} w}{q_{Cu}}, \quad q_{Cu} = \frac{k_{Cu} S_w}{w}, \; w = \frac{E}{\sqrt{2}\pi \hat{B} f A_{Fe} 10^{-4}},$$

it follows that

$$P_{Cu} = \frac{\rho_{Cu} l_{Cu} E^2 10^8 I_{ef}^{\,2}}{\pi^2 k_{Cu} S_w f^2 \hat{B}^2 A_{Fe}^{\,2}}$$

or

(3) $$P_{Cu} = \frac{\rho_{Cu} l_{Cu} P_{tr}^{\,2} 10^8}{\pi^2 k_{Cu} S_w f^2 \hat{B}^2 A_{Fe}^{\,2}}.$$

The core losses are determined by the formula

(4) $$P_{\mathrm{Fe}} = V_{\mathrm{Fe}} g_{\mathrm{Fe}} P_{\mathrm{dFe}} \, .$$

Because of the fact that the specific losses are virtually proportional to $\hat{B}^2$, it follows that

(5) $$P_{\mathrm{dFe}} = \frac{P_{\mathrm{dFe0}} \hat{B}^2}{\hat{B}_0^{\,2}} \, ,$$

where $\hat{B}_0$ and P_{dFe0} are magnetic induction value and the corresponding specific core losses.

When $V_{\mathrm{Fe}} = A_{\mathrm{Fe}} l_{\mathrm{Fe}}$ and P_{dFe0} are replaced correspondingly in (4) and (5) it becomes clear for the core's losses

(6) $$P_{\mathrm{Fe}} = A_{\mathrm{Fe}} l_{\mathrm{Fe}} g_{\mathrm{Fe}} \frac{P_{\mathrm{dFe0}} \hat{B}^2}{\hat{B}_0^{\,2}} \, .$$

The further analysis focuses on small transformers with lamellas, manufactured according to a waste-free technology, and which are also widely applied in our country. The type and markings of such lamellas is shown in Fig. 1.

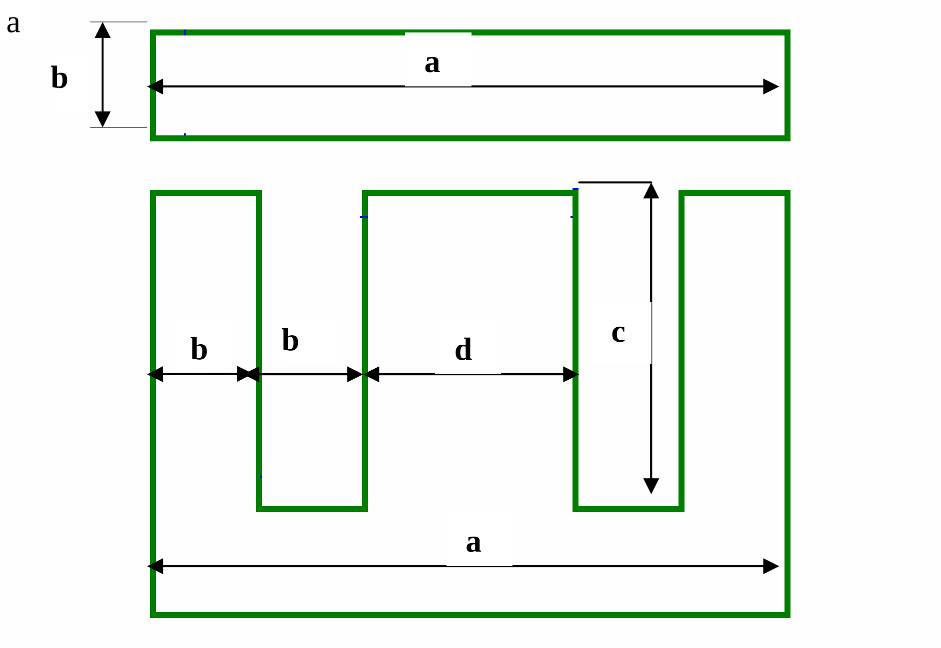

Fig. 1

The goal, which the present article strives to achieve is to determine the core composition at which the total transformers losses (P_{tot}) are minimal, for a given lamella, allowing the construction of a core.

From Fig. 1 it follows that:

$$A_{\mathrm{Fe}} = k_{\mathrm{c}} dx \, , \quad l_{\mathrm{Cu}} = (4d + 2x)10^{-2} \, ,$$

where A_{Fe} is in cm^2, l_{Cu} – in m, and d, x are in cm.

Then, if the core composition is marked with “x”, from (3) and (6) it turns out:

(7) $$P_{\mathrm{Cu}} = \frac{\rho_{\mathrm{Cu}}(4d+2x)P_{\mathrm{tr}}{}^2 10^6}{\pi^2 k_{\mathrm{Cu}} S_{\mathrm{w}} f^2 \hat{B}^2 k_{\mathrm{c}}{}^2 d^2 x^2};$$

(8) $$P_{\mathrm{Fe}} = k_{\mathrm{c}} d x l_{\mathrm{Fe}} g_{\mathrm{Fe}} \frac{P_{\mathrm{dFe0}} \hat{B}^2}{\hat{B}_0{}^2}.$$

After substitution of (7) and (8) in (1), for the total losses, the following formula emerges:

(9) $$P_{\mathrm{tot}} = \frac{2\rho_{\mathrm{Cu}} P_{\mathrm{tr}}{}^2 10^6}{\pi^2 k_{\mathrm{Cu}} S_w f^2 \hat{B}^2 k_{\mathrm{c}}{}^2 d^2}\left(\frac{2d+x}{x^2}\right) + k_{\mathrm{c}} d l_{\mathrm{Fe}} g_{\mathrm{Fe}} \frac{P_{\mathrm{dFe0}} \hat{B}^2}{\hat{B}_0{}^2} x.$$

The minimum of the function $P_{\mathrm{tot}}(x)$ can be precisely found after equalizing the function’s first derivative to zero $\left(\frac{\partial P_{\mathrm{tot}}}{\partial x} = 0\right)$ and the optimal value of “x” is found. After the data from (9) are entered in a mathematical software, in order to examine and for geometrical construction of the function, the minimal losses value and the corresponding size of the core construction are determined.

4. Applying the proposed method to minimize the transformer’s losses

The application of the proposed method will be demonstrated by using a core, applied onto one of the most used lamella – EI96, according to DIN, or YEI-32, according to IEC. The typical dimensions of this lamella, according to Fig. 1 are:

(10) a=9.6 cm; d=3.2 cm; b=1.6 cm; l_{Fe}=19.2 cm; S_{w}=bc=768 mm^2.

Table 1 shows the main parameters of a transformer, applied onto the above-mentioned cold-rolled dynamo sheet M330-35, according to DIN and type-setting 3.4 cm, according to the criteria for achieving maximum power at minimal costs, offered by one of the leading manufacturing companies for small transformers: “Waasner Elektrotechnische Fabrik GmbH” [1].

Table 1

Symbol	P_{tr}, W	$\hat{B}$, T	P_{tot}, W	η, %	Size, cm	m_{Fe}, kg	P_{Fe}, W	P_{Cu}, W	R_{thFe}, °C per 1 W	R_{thCu}, °C per 1 W	ΔT, °C
EI 96, DIN YE1-32, IEC	115	1.44	18.30	84	3.4	1.48	5.50	12.80	2.5	6.1	91.8
Minimal loss transformer	115	1.44	14.09	88	4.0	1.74	6.48	7.61	–	–	58.5

The specific for the given lamella formula of P_{tot} is derived form (9) by replacing the values of the included parameters as follows:

- d, S_{w}, l_{Fe} from (10);
- ρ = 0.0231 (Ω.mm^2)/m for temperature of heating 100 °C;

- f =50 Hz; k_{Cu}= 0.15 (for a single winding); k_c=0.93; g_{Fe}=7.65.10^{-3} W/kg;
- P_{tr}=115 W, $\hat{B}_0 = 1.44$ T, $P_{dFe0} = \dfrac{P_{Fe0}}{m_{Fe}} = \dfrac{5.5}{1.48} = 3.7$, W / kg **(**from Table 1**).**

Then

(11)
$$P_{tot} = \frac{2\times 0.0231\times 115^2\times 10^6}{\pi^2 0.15\times 768\times 50^2 \hat{B}^2 0.93^2\times 3.2^2}.\frac{2\times 3.2 + x}{x^2} + $$
$$+0.93\times 3.2\times 19.2\times 7.65\frac{3.7\hat{B}^2}{1.44^2}x$$

or

(12)
$$P_{tot} = \frac{24.3}{\hat{B}^2}\left(\frac{6.4}{x^2}+\frac{1}{x}\right)+0.78\hat{B}^2 x\,.$$

The value 1.44 T is accepted for $\hat{B}$**,** according to the data in Table 1, which actually represents the maximum acceptable induction for the given core and the material, it is made of. Lowering the $\hat{B}$ values would lead to significant decrease of the transformer power and at the same time to little decrease of the losses.

If $\hat{B}$ = 1.44 T now the equation (12) looks like that

(13)
$$P_{tot} = \left(\frac{75}{x^2}+\frac{11.7}{x}\right)+1.62x.$$

The results of the program analysis of the function $P_{tot}(x)$ from (13) are represented numerically in (14) and graphically in Fig. 2. The values which come out for the losses' minimum and the core's size are

(14) P_{tot}= 13.44 W, x = 5.05 cm.

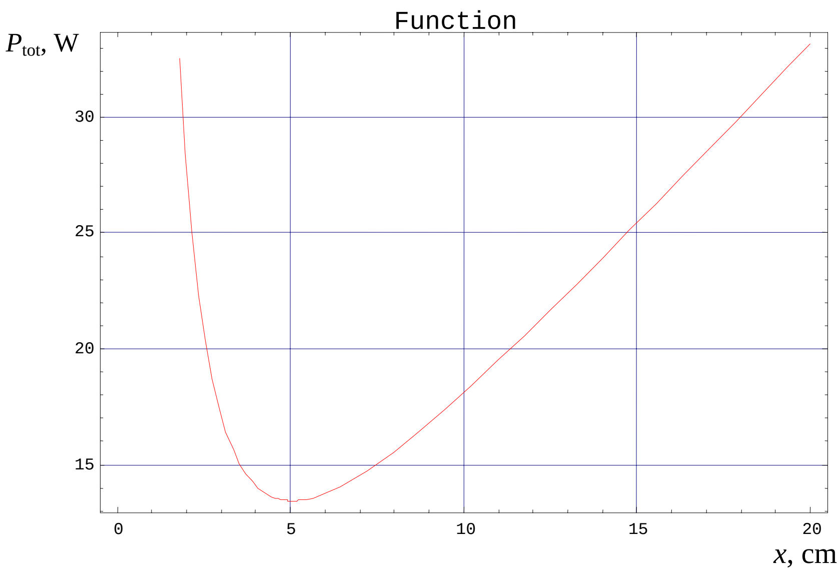

Fig. 2

The minimum of the curve $P_{\text{tot}}(x)$ (Fig. 2) has a pronounced flat region for “*x*” in the region of 4 to 6 mm, because of which, in order to narrow the use of active materials, in the nature of compromise, the value of the core’s size is accepted for *d* = 4 cm.

It follows from (13) that

(15) $$P_{\text{Cu}} = (\frac{75}{4.0^2} + \frac{11,7}{4.0}) = 7.61 \text{ W}; \quad P_{\text{Fe}} = 1.62 \times 4.0 = 6.48 \text{ W}.$$

The values of the parameters of the suggested parameters are also compared in table 1.

Transformer’s overheating temperature is determined via the equation

(16) $$\Delta T = P_{\text{Cu}} R_{\text{thCu}} + P_{\text{Fe}} R_{\text{thFe}} .$$

In the suggested transformer, the same values of R_{thCu} and R_{thFe} are used as in the base transformer.

Table 1 shows that the total losses P_{tot} have decreased from 18.3 to 14.09 W, i.e. with 30% and the efficiency has been increased from 84 to 88%.

Furthermore, the overheating temperature of the base transformer is $\Delta T = 5.5 \times 2.5 + 12.8 \times 6.1 = 91.8\,^{\circ}\text{C}$ while the overheating temperature of the suggested transformer is $\Delta T = 7.61 \times 2.5 + 6.48 \times 6.1 = 58.55\,^{\circ}\text{C}$, which will lead to increasing of the reliability of the transformer, and consequently to an increase of the reliability of the device this transformer would be eventually installed.

The assessment of the introduction of low loss transformers depends on the individual application. A compromise needs to be done between the lowering of the losses and the increase in the price. The time, for which the higher price, cased by the low losses, could be paid off, may be used as an evaluation criteria.

5. Conclusion

- This article offers a method for the application of small line power frequency transformers with minimized losses, the core of which is made of silicon steel.
- It defines how the total losses of the small transformer depend on the power of the transformer and on the geometry of the core. In this case a small transformer with a core, constructed realized by waste-free, EI lamellas has been analyzed. The same method could be also applied on lamella cores, realized by a small-waste technology and on winded cores.
- The results of the program analysis are represented numerically and graphically and the values for the losses’ minimum and the corresponding size of the core have been determined.
- It is proven by a specific example (EI 96 lamella) that the application of the suggested method lowers the total costs in the transformer by about 30%.

R e f e r e n c e s

1. Waasner Elektrotechnische Fabrik GmbH, Elektro-Bauteile-Katalog.

Сетевые трансформаторы малой мощности с минимизированными потерями

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(Р е з ю м е)

В статье предложен подход расчета сетевых трансформаторов малой мощности с минимизированными потерями. Определена зависимость суммарных потерь в зависимости от мощности трансформаторов, от параметров материала и размеров сердечника. Результаты программного анализа представлены в числовом и графическом виде, при чем определен минимум потерь для конкретного применения.