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Multicriteria Decision Making Based on Fuzzy Relations

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Abstract*. The considered models describe multicriteria decision making when the information provided from different criteria is given as fuzzy relations. These models differ according to the kind of the criteria weights and the aggregation procedures to fuse the fuzzy relations. The properties of aggregated fuzzy relation required to decide some multicriteria problem is discussed.*

Keywords*: Aggregation of fuzzy relations, weigths of criteria, t-norms, t-conorms, composition of fuzzy relations, properties of fuzzy relations.*

1. Introduction

The models of the multicriteria decision making are based on:

- finite set of alternatives,
- finite set of criteria (properties, experts) that evaluate the alternatives,
- weights (importances) of the criteria.

The **alternatives** are usually evaluated from different point of view according to individual criteria. In the real situations, the evaluations are inaccurate, uncertain and fuzzy in most cases. This shows the necessity of using the fuzzy sets theory taking a possibility to make decisions as close as possible to humans' decision making.

The **criteria** can be quantitative, qualitative, fuzzy or mixed. The quantitative criteria give generally accurate numerical evaluations. The qualitative criteria evaluate the alternatives by qualitative variables that have a natural order and the fuzzy criteria give as evaluations or fuzzy numbers for each alternative or fuzzy relations between the couple of alternatives.

The **weights** of the criteria can be:

- normalized real numbers with sum equal to one,

- weighting functions depending on the membership degrees of the fuzzy relations,
- fuzzy numbers,
- fuzzy relation between the criteria importances.

The weights or the importances of the criteria are chosen from the experts or the decision maker. Procedures for determining the weights have been the aim of many investigations and discussions [1, 26, 49, 31, 50, 54, 60].

The **aims** of the multicriteria decision making problems are:

- choice of a subset from the best, in some sense, alternatives,
- ordering of the alternatives from the best to the worst,
- clustering the set of alternatives.

These aims can be reached by comparing the couple of alternatives according to the:

- type of the criteria,
- importances (weights) of the criteria,
- possible difficulties comparing two alternatives, e.g. when the one of them is better according to subset of criteria, but worse at least for one criterion of the complementary set.

Let $A=\{a_1,...,a_i,...,a_n\}$ be a finite set of alternatives evaluated by criteria $K=\{k_1,...,k_j...,k_m\}$ with weights $W=\{w_1,...,w_j,...,w_m\}$. The model can be written in a matrix form as shown in Table 1, where x_{ij}, $i=1,...,n, j=1,...,m$, is the

Table 1

Alternatives	Criteria				
	k_1	…	k_j	…	k_m
a_1	x_{11}	…	x_{1j}	…	x_{1m}
⋮	⋮	⋮	⋮	⋮	⋮
a_i	x_{i1}	…	x_{ij}	…	x_{im}
⋮	⋮	⋮	⋮	⋮	⋮
a_n	x_{n1}	…	x_{nj}	…	x_{nm}

evaluation of a_i by criterion k_j. As it was said before these values can be numerical, qualitative variables and fuzzy ones. This information has to be unified. A basic method to perform this is: to transform information into fuzzy relations by comparing the couples of the alternatives' evaluations [7, 37, 39, 52]. When the number of alternatives n of A is not very large, then the fuzzy relation R_k, corresponding to the criterion k_k can be presented as a ($n \times n$) matrix $R_k = \left\| r_{ij}^k \right\|$, where $r_{ij}^k = \mu_k(a_i, a_j), i, j=1, ..., n, k=1, ..., m$, $\mu_k : A \times A \to [0, 1]$ is the membership function of the relation R_k and r_{ij}^k is the membership degree to R_k

obtained by comparison of the couple of alternatives a_i and a_j according to criterion k_k, i. e.

$$R_k = \begin{pmatrix} \mu_k(a_1,a_1)\dots & \mu_k(a_1,a_j) & \dots & \mu_k(a_1,a_n) \\ \vdots & \vdots & & \vdots \\ \mu_k(a_i,a_1)\dots & \mu_k(a_i,a_j) & \dots & \mu_k(a_i,a_n) \\ \vdots & \vdots & \dots & \vdots \\ \mu_k(a_n,a_1) & \mu_k(a_n,a_j) & \dots & \mu_k(a_n,a_n) \end{pmatrix} =$$

$$= \begin{pmatrix} r_{11}^k & \dots & r_{1j}^k & \dots & r_{1n}^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{i1}^k & \dots & r_{ij}^k & \dots & r_{in}^k \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ r_{n1}^k & \dots & r_{nj}^k & \dots & r_{nn}^k \end{pmatrix}, k = 1,\dots,m.$$

The obtained fuzzy relations by all criteria have to be fused into a single fuzzy relation. This can be performed using aggregation procedures that compensate the conflict criteria by compromise. The resulting compromise has to be located between the most optimistic lower limit and the most pessimistic upper limit of the estimations by the criteria, i.e. between the maximal and minimal membership degrees of the relations. The most preferable procedures are the ones that use aggregation operators. The survey of these operators is presented in [13, 25], where their characteristics, advantages, faults and the relationships between them are presented. The properties of the aggregated relation depending on the properties of all particular relations and depending on the used aggregation operator as well are proved and systematized in [15, 16, 35, 36, 38, 39, 40, 42].

There exist many operators which can be used for the aims of the aggregation. The choice of a given operator depends on different factors [19], e.g.:

- the estimations' kind,
- the mathematical model of the operator,
- the properties of the operator appropriated for solving the problems of a choice, an ordering or a clustering of the alternatives' set,
- the sensitivity of the operators by small variations of their arguments [41].

2. Models for criteria with weights real numbers

When the weights are given as real numbers the operators: Weighted Mean [3, 4, 7, 58, 59], Weighted Geometric [8]; Weighted MaxMin and Weighted MinMax [22], e.g. can be used for aggregation of fuzzy relations. There are the operators in the mathematical model of which the weights do not present, e.g. the operators: Min, Max, MaxMin [53], Gamma [53], Generalized Mean [13, 25, 51]. An idea to use the given weights in this case is suggested in [57].

Let $\mu_k(a_i, a_j), k = 1,...m, \forall a_i, a_j \in A,$ be the membership degree to the fuzzy relation R_k and $\mu(a_i, a_j), \forall a_i, a_j \in A,$ be the membership degree to the aggregated relation. The given vector of the weight coefficients is $W = \{w_1,..., w_j,..., w_m\}$. The membership degrees can be transformed taking into accout the weight coefficients by the following way:

$$g(w_k, \mu_k(a_i, a_j)) = \mu_k^w(a_i, a_j), \; a_i, a_j \in A, k = 1,..., m,$$

where the function g possesses the properties:

- $g(w, x)$ is monotonic for both arguments,
- $g(0, x) = \text{id}, g(1, x) = x$, id is the identical element.

It is known that the *t*-norms and *t*-conorms possess these properties. In performing the Min aggregation there are elements with small values that play the most significant role in this type of aggregation. One way to reduce the effect of elements with low importance is to transform them into values closer to one. Yager introduces a class of functions that can be used for the inclusion of weights in the Min aggregation

$$g(w_k, \mu_k(a_i, a_j)) = S(1 - w_k, \mu_k(a_i, a_j)),$$

where S is a *t*-conorm, and then

$$\min\{\mu_1^w(a_i, a_j),..., \mu_m^w(a_i, a_j)\} =$$
$$= \min\{S(1 - w_1, \mu_1(a_i, a_j)),..., S(1 - w_m, \mu_m(a_i, a_j))\}.$$

It is obvious that if $w_k = 0$, then $S(1 - w_k, \mu_k(a_i, a_j)) = S(1, \mu_k(a_i, a_j)) = 1$ and this element plays no pole in the Min aggregation. Yager notes that this formulae can be seen as a measure of the degree to which an aggregation estimation satisfies the following proposition "All important criteria are satisfied".

In performing the Max aggregation the transformation

$$g(w_k, \mu_k(a_i, a_j)) = T(w_k, \mu_k(a_i, a_j))$$

may be used, where T is a *t*-norm and then

$$\max\{\mu_1^w(a_i, a_j),..., \mu_m^w(a_i, a_j)\} = \max\{T(w_1, \mu_1(a_i, a_j)),..., T(w_m, \mu_m(a_i, a_j))\}.$$

If $w_k = 0$, then $T(w_k, \mu_k(a_i, a_j)) = 0$ and the element plays no role in the aggregation.

Then the arguments of the aggregation operator Agg will be the transformed membership degrees:

$$\text{Agg}(\mu_1^w(a_i, a_j),..., \mu_m^w(a_i, a_j)) = \mu^w(a_i, a_j).$$

The properties of the aggregation operators with transformed membership degrees are investigated in [43, 44].

3. Models for criteria with weights real functions (weighting functions)

The idea of considering weighting functions that depend continuously on the criterion satisfaction values (i.e. good or bad criteria performances) is supported by common sense reasoning and experience in the context of decision theory [31, 50]. From a decision maker point of view, when a single alternative is considered, the weight of an important criterion with a low satisfaction value should be finable in some cases, in order to render the given criterion less significant in the overall evaluation of the alternative. Accordingly, when two alternatives are considered, the dominance effect in one important criterion would become less significant, when the criterion satisfaction values are low. The weight of a less important criterion with higher satisfaction values should in some cases be rewarded to render more significant the dominance effect in that criterion. That's way, it may be relevant to use weights depending on the criterion satisfaction values.

The introduction of weighting functions depending continuously on criterion satisfaction values produces weighted aggregation operators with complex dependency from these values. These functions have to be monotonic and sensitive [31], as well. Monotonocity requires that the aggregation operator's value has to increase, when any of the criterion satisfaction values increases. The central question is that monotonicity involves constraints on derivatives of the weighted aggregation operator with respect to the various criterion satisfaction values. There is also a question of sensitivity, involving constraints on derivatives of the weighted aggregation operator with respect to the various numerical weights. Sensitivity requires that the relative contribution of the criterion to the value of the aggregation operator increases when the corresponding weight increases.

Let $f_1(x),...,f_m(x), x \in [0,1]$ be some real functions. As was said in Section 2, the membership degrees can be transformed using the weighting functions by the following way:

$$\mu_k^w(a_i, a_j) = g(w_k(\mu_k(a_i, a_j)), \mu_k(a_i, a_j))$$
$$\forall a_i, a_j \in A, k = 1,...,m, \ i, j = 1,...,n,$$

where $w_k(\mu_k(a_i, a_j)) \in [0,1]$ is the weighting coefficient computed with the help of the function $f_k(x)$ and g(*w, x*) is some *t*-norm or *t*-conorm. Then the membership of the aggregated relation will be:

$$\mu^w(a_i, a_j) = \text{Agg}(\mu_1^w(a_i, a_j),...,\mu_m^w(a_i, a_j)),$$

where Agg denotes some aggregation operator.

The following fitting [32] weighting functions are considered in [31]:

- linear weighting functions

  $f_k(x) = 1 + \beta_k x$ with parameters $0 \le \beta_k \le 1, \ k = 1, ..., m, \ m \ge 2$;

- parametric linear weighting functions

$$f_k(x) = \alpha_k \frac{1 + \beta_k x}{1 + \beta_k} = \gamma_k (1 + \beta_k x),$$

$$0 < \alpha_k \le 1, \ 0 \le \beta_k \le 1, \ \gamma_k = \frac{\alpha_k}{1 + \beta_k}, \ k = 1,...,m;$$

- quadratic weighting functions

$$f_k(x) = 1 + (\beta_k - \gamma_k)x + \gamma_k x^2,\ \beta_k \geq 0,\ \gamma_k \geq 0,\ k = 1,...,m.$$

The continuous functions $f_k(x),\ k = 1,...,m,$ are defined in the unit interval for each $x \in [0, 1]$ and they have continuous derivatives $f_k\,'(x), k = 1,...,m,$ in this interval.

Let the membership degrees comparing the alternatives $a_i,\ a_j \in A$ to fuzzy relations $R_1,..., R_k,..., R_m$ are $\mu_1(a_i, a_j) = x_{ij}^1,\ ..., \mu_k(a_i, a_j) = x_{ij}^k,\ \mu_m(a_i, a_j) = x_{ij}^m$. The Generalized mixture operator with the following mathematical model is defined in [31]:

$$\mu^w(a_i, a_j) = \mathrm{Agg}(\mu_1^w(a_i, a_j),..., \mu_m^w(a_i, a_j)) =$$

$$= \begin{cases} 1 & \text{if } a_i = a_j \\ \dfrac{\sum_{k=1}^{m} f_k(x_{ij}^k) x_{ij}^k}{S(a_i, a_j)} & \text{if } a_i \neq a_j \end{cases},\ k = 1,...,m,\ i, j = 1,...,n,$$

where $S(a_i, a_j) = \sum_{k=1}^{M} f_k(x_{ij}^k)$, and $f_k(\bullet)$ is one of the weighting functions given above. It is proved [31] that the sufficient condition for strict monotonicity of the Generalized mixture operator, i.e. to be aggregation operator is:

$$0 \leq f_k\,'(x) \leq f_k(x),\ k = 1,...,m,\ x \in [0, 1].$$

In this case the properties of the aggregated relations obtained by the Generalized mixture operator taking into account the above weighting functions are investigated and proved in [46, 47, 48].

4. Models for criteria with fuzzy weights

Fuzzy weights indicate that the weights are fuzzy numbers or a fuzzy relation between the importances of the criteria. Two ways to solve the case when the weights are fuzzy numbers are available till now. The first way uses some ranking function. Many authors follow this approach, e.g. [2, 5, 6, 14, 18, 20, 21, 24, 27, 29, 36, 56]. The idea of these methods is the following. Let $\tilde{A}_i,\ i = 1,...,n$, are normal, convex fuzzy sets (fuzzy numbers), i.e.

$$\tilde{A}_i = \{x, \mu_i(x)\},\ x \in I_i \subset I,\ I \equiv [0, 1],$$

with the membership function $\mu_i(x)$. A ranking function *F*, that maps each fuzzy number into real line, is defined and if $F(\tilde{A}_i) <=> F(\tilde{A}_j)$, then $\tilde{A}_i <=> \tilde{A}_j$.

The second approach computes an index comparing each couple of fuzzy numbers. A fuzzy relation is obtained as a result of this comparison. This approach is used, e.g. in [9, 10, 11, 12, 23, 28, 30, 33, 34, 61].

If the ranking function is used then each fuzzy number is reduced into a real number and therefore the problem is the same as the one considered in section 2.

If the fuzzy relation between the fuzzy numbers is obtained the following approach can be used.

Let $A = \{a_1,...,a_n\}$ be the alternatives' set, $K = \{k_1,...,k_m\}$ be the set of criteria, which give the fuzzy relations $R_1, R_2,...,R_m$ between the alternatives. The relations can be represented as $n \times n$ matrix $R_k = \left\| r_{ij}^k \right\|$, where $r_{ij}^k = \mu_k(a_i, a_j),\ i, j = 1,...,n,\ k = 1,...,m,$ is the preference degree of the alternative a_i to a_j according to criterion k_k. If $r_{ij}^k = 0.5$ this indicates indifference between a_i and a_j, and if $r_{ij}^k = 1$ then by this criterion the alternative a_i is absolutely preferred to a_j and $r_{ij}^k > 0.5$ indicates that a_i are preferred to a_j by the *k*-th criterion. In this case it is supposed that the matrices $R_k,\ k = 1,...,m,$ are additive reciprocal and max-min transitive, i.e. respectively

$$r_{ij}^k + r_{ji}^k = 1, i, j = 1,...,n, \text{ and } r_{ij}^k \geq \min(r_{is}^k, r_{si}^k),\ i, j, s = 1,...,n .$$

The fuzzy preference relation *W* between the importances of the criteria is given as well, i.e. the couples of criteria are compared by their importances and the preference degrees are determined. Let $W = \left\| w(k_i, k_j) \right\| = \left\| w_{ij} \right\|,\ i, j = 1, ..., m,$ where $w(k_i, k_j)$ is the preference degree of the criterion k_i to k_j. The procedure includes these preference degrees into the aggregation process using the transformed preference degree r_{ij}^k by w_{ij}. The transformation may be performed with the help of considered functions *g* in Section 2 together with the corresponding properties. Examples for such kind of functions are: Minimum operator [57], an exponential function [55] and one of the *t*-norms [62, 44].

In this sense the properties of the operation composition between two fuzzy relations are investigated [45]. It is proved that the composition is max-Δ transitive, i.e.

$$r_{ij}^k \geq \max(0,\ r_{is}^k + r_{si}^k - 1),\ i, j, s = 1,...,n ,$$

when the two relations are additive reciprocal and max-min transitive. A combination of *t*-norm and corresponding *t*-conorm is used to obtain a new relation including the composition. It is proved that this new relation preserves the property of max-Δ transitivity of the composition by some determined conditions, which are requited to obtain an alternatives' order. The properties of the aggregated fuzzy relation are determined and proved.

5. Conclusion

Models of the multicriteria decision making by fuzzy criteria are considered. These criteria give fuzzy relations by each criterion. Different weights of the criteria are used in the aggregation process: weighting coefficients, weighting functions, fuzzy numbers and fuzzy preference relation between the criteria importances. The aggregation procedures depend on the kind of the criteria weights. The properties of the aggregated relation are determined and proved. These properties are required to solve the problems of: a choice of some subset of preferred alternatives, ordering or clustering of the alternatives' set.

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